

# Nombres heureux

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Si  $n$  est un entier naturel, on note  $S(n)$  la somme des carrés de ses chiffres.

Par exemple,  $S(21) = 2^2 + 1^2 = 5$ . On peut très bien recommencer l'opération avec le dernier nombre obtenu :  $S(5) = 5^2 = 25$ ,  $S(25) = 2^2 + 5^2 = 29$ , etc. ce qui donne la suite :

$$21, 5, 25, 29, \dots$$

L'objet de cet article est de démontrer le résultat suivant :

## Théorème

Quelque soit le nombre entier strictement positif de départ, en calculant successivement la somme des carrés des chiffres, on atteint nécessairement 1 ou 4.

Par exemple, si on part de  $n = 34$ , on atteint 1 en quatre étapes :

$$19, 82, 68, 100, 1, \dots$$

Autre exemple : si on part de  $n = 24$ , on atteint 4 en deux étapes :

$$24, 20, 4, \dots$$

## 1 Résultats préliminaires

### Propriété 1

Soit  $n > 0$  un entier naturel. Si  $m$  possède  $m$  chiffres dans son écriture en base 10, alors la somme des carrés de ses chiffres est inférieure ou égale à  $81m$ .

*Démonstration.* Puisque  $n$  possède  $m$  chiffres, on peut écrire  $n = a_0 \times 10^0 + a_1 \times 10^1 + \dots + a_m \times 10^m$ . La somme des carrés de ses chiffres est donc :

$$a_0^2 + a_1^2 + \dots + a_m^2$$

Comme, chaque chiffre  $a_i$  est inférieur ou égal à 9, on a :

$$a_0^2 + a_1^2 + \dots + a_m^2 \leq \underbrace{9^2 + 9^2 + \dots + 9^2}_{m \text{ fois}} = 9^2 \times m = 81m$$

□

### Propriété 2

Pour tout entier naturel  $m \geq 4$ ,

$$81m \leq 10^{m-1}$$

*Démonstration.* Montrons ce résultat par récurrence.

- $81 \times 4 = 324$  et  $10^{4-1} = 100$  donc la propriété est vraie pour  $m = 4$ .
- Soit  $m \geq 4$  et supposons que  $81m \leq 10^{m-1}$ . Notons tout d'abord que comme  $1 \leq m$ , on a  $81 \leq 81m$  donc  $81 \leq 10^{m-1}$ . De plus,

$$81(m+1) = 81m + 81 \leq 10^{m-1} + 10^{m-1} = 2 \times 10^{m-1} \leq 10 \times 10^{m-1} = 10^m$$

donc  $81(m+1) \leq 10^m$ . □

### Propriété 3

Soit  $n \geq 1000$ .

La somme des carrés des chiffres de  $n$  est strictement inférieure à  $n$ .

*Démonstration.* D'après la propriété 1,  $S(n)$  (la somme des carrés des chiffres de  $n$ ) est inférieure ou égale à  $81m$  où  $m$  est le nombre de chiffres de  $n$ . Comme  $n \geq 1000$ , alors  $n$  possède au moins 4 chiffres, c'est-à-dire  $m \geq 4$ . D'après la propriété précédente, on peut donc affirmer que la somme des carrés des chiffres de  $n$  est inférieure ou égale à  $10^{m-1}$  ( $S(n) \leq 10^{m-1}$ ).

Or, si  $n = a_0 \times 10^0 + a_1 \times 10^1 + \dots + a_m \times 10^m$  alors  $n \geq a_m 10^m \geq 10^m$  (car  $a_m \geq 1$ ) donc

$$S(n) \leq 10^{m-1} < 10^m \leq n$$

d'où le résultat. □

### Propriété 4

Soit  $n \geq 1000$ .

En calculant successivement la somme des carrés des chiffres en partant de  $n$ , on atteint nécessairement un nombre inférieur ou égal à 1000 à un certain rang.

*Démonstration.* Soit  $n \geq 1000$ . Supposons que la suite des sommes des carrés des chiffres soit toujours strictement supérieure à 1000. On peut alors dire que cette suite est strictement décroissante d'après la propriété précédente et donc, puisque c'est une suite d'entiers naturels, elle est stationnaire à partir d'un certain rang. Autrement dit, il existerait un  $n_0 > 1000$  (puisque la suite est supposée être toujours strictement supérieure à 1000) tel que  $S(n_0) = n_0$ , ce qui est impossible si  $n_0 \geq 1000$  d'après la propriété précédente.

Il existe donc bien un rang pour lequel la suite prend une valeur inférieure ou égale à 1000. □

### Propriété 5

En calculant successivement la somme des carrés des chiffres en partant d'un nombre compris entre 1 et 1000, on atteint nécessairement soit 1, soit 4 à partir d'un certain moment.

*Démonstration.* La preuve est une « simple » étude des mille cas possibles. Les résultats sont présentés dans les pages suivantes. Ils ont été obtenus à l'aide de l'outil informatique. □

1	1, 1	51	51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	101	101, 2, 4	151	151, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	201	201, 5, 25, 29, 85, 89, 145, 42, 20, 4
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265	265, 65, 61, 37, 58, 89, 145, 42, 20, 4	315	315, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	365	365, 70, 49, 97, 130, 10, 1	415	415, 42, 20, 4	465	465, 77, 98, 145, 42, 20, 4
266	266, 76, 85, 89, 145, 42, 20, 4	316	316, 46, 52, 29, 85, 89, 145, 42, 20, 4	366	366, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	416	416, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	466	466, 88, 128, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
267	267, 89, 145, 42, 20, 4	317	317, 59, 106, 37, 58, 89, 145, 42, 20, 4	367	367, 94, 97, 130, 10, 1	417	417, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	467	467, 101, 2, 4
268	268, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	318	318, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	368	368, 109, 82, 68, 100, 1	418	418, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	468	468, 116, 38, 73, 58, 89, 145, 42, 20, 4
269	269, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	319	319, 91, 82, 68, 100, 1	369	369, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	419	419, 98, 145, 42, 20, 4	469	469, 133, 19, 82, 68, 100, 1
270	270, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	320	320, 13, 10, 1	370	370, 58, 89, 145, 42, 20, 4	420	420, 20, 4	470	470, 65, 61, 37, 58, 89, 145, 42, 20, 4
271	271, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	321	321, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	371	371, 59, 106, 37, 58, 89, 145, 42, 20, 4	421	421, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4	471	471, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
272	272, 57, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	322	322, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	372	372, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	422	422, 24, 20, 4	472	472, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
273	273, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	323	323, 22, 8, 64, 52, 29, 85, 89, 145, 42, 20, 4	373	373, 67, 85, 89, 145, 42, 20, 4	423	423, 29, 85, 89, 145, 42, 20, 4	473	473, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
274	274, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	324	324, 29, 85, 89, 145, 42, 20, 4	374	374, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	424	424, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	474	474, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
275	275, 78, 113, 11, 2, 4	325	325, 38, 73, 58, 89, 145, 42, 20, 4	375	375, 83, 73, 58, 89, 145, 42, 20, 4	425	425, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	475	475, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
276	276, 89, 145, 42, 20, 4	326	326, 49, 97, 130, 10, 1	376	376, 94, 97, 130, 10, 1	426	426, 56, 61, 37, 58, 89, 145, 42, 20, 4	476	476, 101, 2, 4
277	277, 102, 5, 25, 29, 85, 89, 145, 42, 20, 4	327	327, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	377	377, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4	427	427, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	477	477, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
278	278, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	328	328, 77, 98, 145, 42, 20, 4	378	378, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	428	428, 84, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	478	478, 129, 86, 100, 1
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280	280, 68, 100, 1	330	330, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	380	380, 73, 58, 89, 145, 42, 20, 4	430	430, 25, 29, 85, 89, 145, 42, 20, 4	480	480, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4
281	281, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	331	331, 19, 82, 68, 100, 1	381	381, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	431	431, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	481	481, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
282	282, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	332	332, 22, 8, 64, 52, 29, 85, 89, 145, 42, 20, 4	382	382, 77, 98, 145, 42, 20, 4	432	432, 29, 85, 89, 145, 42, 20, 4	482	482, 84, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4
283	283, 77, 98, 145, 42, 20, 4	333	333, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	383	383, 82, 68, 100, 1	433	433, 34, 25, 29, 85, 89, 145, 42, 20, 4	483	483, 89, 145, 42, 20, 4
284	284, 84, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	334	334, 34, 25, 29, 85, 89, 145, 42, 20, 4	384	384, 89, 145, 42, 20, 4	434	434, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	484	484, 96, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
285	285, 93, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	335	335, 43, 25, 29, 85, 89, 145, 42, 20, 4	385	385, 98, 145, 42, 20, 4	435	435, 50, 25, 29, 85, 89, 145, 42, 20, 4	485	485, 105, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
286	286, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	336	336, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	386	386, 109, 82, 68, 100, 1	436	436, 61, 37, 58, 89, 145, 42, 20, 4	486	486, 116, 38, 73, 58, 89, 145, 42, 20, 4
287	287, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	337	337, 67, 85, 89, 145, 42, 20, 4	387	387, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	437	437, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	487	487, 129, 86, 100, 1
288	288, 132, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	338	338, 82, 68, 100, 1	388	388, 137, 59, 106, 37, 58, 89, 145, 42, 20, 4	438	438, 89, 145, 42, 20, 4	488	488, 144, 33, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
289	289, 149, 98, 145, 42, 20, 4	339	339, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	389	389, 154, 42, 20, 4	439	439, 106, 37, 58, 89, 145, 42, 20, 4	489	489, 161, 38, 73, 58, 89, 145, 42, 20, 4
290	290, 85, 89, 145, 42, 20, 4	340	340, 25, 29, 85, 89, 145, 42, 20, 4	390	390, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	440	440, 32, 13, 10, 1	490	490, 97, 130, 10, 1
291	291, 86, 100, 1	341	341, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	391	391, 91, 82, 68, 100, 1	441	441, 33, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	491	491, 98, 145, 42, 20, 4
292	292, 89, 145, 42, 20, 4	342	342, 29, 85, 89, 145, 42, 20, 4	392	392, 94, 97, 130, 10, 1	442	442, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	492	492, 101, 2, 4
293	293, 94, 97, 130, 10, 1	343	343, 34, 25, 29, 85, 89, 145, 42, 20, 4	393	393, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	443	443, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	493	493, 106, 37, 58, 89, 145, 42, 20, 4
294	294, 101, 2, 4	344	344, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	394	394, 106, 37, 58, 89, 145, 42, 20, 4	444	444, 48, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	494	494, 113, 11, 2, 4
295	295, 110, 2, 4	345	345, 50, 25, 29, 85, 89, 145, 42, 20, 4	395	395, 115, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	445	445, 57, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	495	495, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
296	296, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	346	346, 61, 37, 58, 89, 145, 42, 20, 4	396	396, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	446	446, 68, 100, 1	496	496, 133, 19, 82, 68, 100, 1
297	297, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	347	347, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	397	397, 139, 91, 82, 68, 100, 1	447	447, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	497	497, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
298	298, 149, 98, 145, 42, 20, 4	348	348, 89, 145, 42, 20, 4	398	398, 154, 42, 20, 4	448	448, 96, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	498	498, 161, 38, 73, 58, 89, 145, 42, 20, 4
299	299, 166, 73, 58, 89, 145, 42, 20, 4	349	349, 106, 37, 58, 89, 145, 42, 20, 4	399	399, 171, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20,				

501	501, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	551	551, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	601	601, 37, 58, 89, 145, 42, 20, 4	651	651, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	701	701, 50, 25, 29, 85, 89, 145, 42, 20, 4
502	502, 29, 85, 89, 145, 42, 20, 4	552	552, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	602	602, 40, 16, 37, 58, 89, 145, 42, 20, 4	652	652, 65, 61, 37, 58, 89, 145, 42, 20, 4	702	702, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
503	503, 34, 25, 29, 85, 89, 145, 42, 20, 4	553	553, 59, 106, 37, 58, 89, 145, 42, 20, 4	603	603, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	653	653, 70, 49, 97, 130, 10, 1	703	703, 58, 89, 145, 42, 20, 4
504	504, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	554	554, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	604	604, 52, 29, 85, 89, 145, 42, 20, 4	654	654, 77, 98, 145, 42, 20, 4	704	704, 65, 61, 37, 58, 89, 145, 42, 20, 4
505	505, 50, 25, 29, 85, 89, 145, 42, 20, 4	555	555, 75, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	605	605, 61, 37, 58, 89, 145, 42, 20, 4	655	655, 86, 100, 1	705	705, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
506	506, 61, 37, 58, 89, 145, 42, 20, 4	556	556, 86, 100, 1	606	606, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	656	656, 97, 130, 10, 1	706	706, 85, 89, 145, 42, 20, 4
507	507, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	557	557, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	607	607, 85, 89, 145, 42, 20, 4	657	657, 110, 2, 4	707	707, 98, 145, 42, 20, 4
508	508, 89, 145, 42, 20, 4	558	558, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	608	608, 100, 1	658	658, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	708	708, 113, 11, 2, 4
509	509, 106, 37, 58, 89, 145, 42, 20, 4	559	559, 131, 11, 2, 4	609	609, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	659	659, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4	709	709, 130, 10, 1
510	510, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	560	560, 61, 37, 58, 89, 145, 42, 20, 4	610	610, 37, 58, 89, 145, 42, 20, 4	660	660, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	710	710, 50, 25, 29, 85, 89, 145, 42, 20, 4
511	511, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	561	561, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	611	611, 38, 73, 58, 89, 145, 42, 20, 4	661	661, 73, 58, 89, 145, 42, 20, 4	711	711, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
512	512, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	562	562, 65, 61, 37, 58, 89, 145, 42, 20, 4	612	612, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	662	662, 76, 85, 89, 145, 42, 20, 4	712	712, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
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514	514, 42, 20, 4	564	564, 77, 98, 145, 42, 20, 4	614	614, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	664	664, 88, 128, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	714	714, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
515	515, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	565	565, 86, 100, 1	615	615, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	665	665, 97, 130, 10, 1	715	715, 75, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
516	516, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4	566	566, 97, 130, 10, 1	616	616, 73, 58, 89, 145, 42, 20, 4	666	666, 108, 65, 61, 37, 58, 89, 145, 42, 20, 4	716	716, 86, 100, 1
517	517, 75, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	567	567, 110, 2, 4	617	617, 86, 100, 1	667	667, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	717	717, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
518	518, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	568	568, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	618	618, 101, 2, 4	668	668, 136, 46, 52, 29, 85, 89, 145, 42, 20, 4	718	718, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
519	519, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4	569	569, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4	619	619, 118, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	669	669, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	719	719, 131, 11, 2, 4
520	520, 29, 85, 89, 145, 42, 20, 4	570	570, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	620	620, 40, 16, 37, 58, 89, 145, 42, 20, 4	670	670, 85, 89, 145, 42, 20, 4	720	720, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
521	521, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	571	571, 75, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	621	621, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	671	671, 86, 100, 1	721	721, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
522	522, 33, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	572	572, 78, 113, 11, 2, 4	622	622, 44, 32, 13, 10, 1	672	672, 89, 145, 42, 20, 4	722	722, 57, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
523	523, 38, 73, 58, 89, 145, 42, 20, 4	573	573, 83, 73, 58, 89, 145, 42, 20, 4	623	623, 49, 97, 130, 10, 1	673	673, 94, 97, 130, 10, 1	723	723, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4
524	524, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	574	574, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	624	624, 56, 61, 37, 58, 89, 145, 42, 20, 4	674	674, 101, 2, 4	724	724, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
525	525, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	575	575, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	625	625, 65, 61, 37, 58, 89, 145, 42, 20, 4	675	675, 110, 2, 4	725	725, 78, 113, 11, 2, 4
526	526, 65, 61, 37, 58, 89, 145, 42, 20, 4	576	576, 110, 2, 4	626	626, 76, 85, 89, 145, 42, 20, 4	676	676, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	726	726, 89, 145, 42, 20, 4
527	527, 78, 113, 11, 2, 4	577	577, 123, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	627	627, 89, 145, 42, 20, 4	677	677, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	727	727, 102, 5, 25, 29, 85, 89, 145, 42, 20, 4
528	528, 93, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	578	578, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	628	628, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	678	678, 149, 98, 145, 42, 20, 4	728	728, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
529	529, 110, 2, 4	579	579, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	629	629, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	679	679, 166, 73, 58, 89, 145, 42, 20, 4	729	729, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
530	530, 34, 25, 29, 85, 89, 145, 42, 20, 4	580	580, 89, 145, 42, 20, 4	630	630, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	680	680, 100, 1	730	730, 58, 89, 145, 42, 20, 4
531	531, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	581	581, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	631	631, 46, 52, 29, 85, 89, 145, 42, 20, 4	681	681, 101, 2, 4	731	731, 59, 106, 37, 58, 89, 145, 42, 20, 4
532	532, 38, 73, 58, 89, 145, 42, 20, 4	582	582, 93, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	632	632, 49, 97, 130, 10, 1	682	682, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	732	732, 62, 40, 16, 37, 58, 89, 145, 42, 20, 4
533	533, 43, 25, 29, 85, 89, 145, 42, 20, 4	583	583, 98, 145, 42, 20, 4	633	633, 54, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	683	683, 109, 82, 68, 100, 1	733	733, 67, 85, 89, 145, 42, 20, 4
534	534, 50, 25, 29, 85, 89, 145, 42, 20, 4	584	584, 105, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	634	634, 61, 37, 58, 89, 145, 42, 20, 4	684	684, 116, 38, 73, 58, 89, 145, 42, 20, 4	734	734, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
535	535, 59, 106, 37, 58, 89, 145, 42, 20, 4	585	585, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	635	635, 70, 49, 97, 130, 10, 1	685	685, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	735	735, 83, 73, 58, 89, 145, 42, 20, 4
536	536, 70, 49, 97, 130, 10, 1	586	586, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	636	636, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	686	686, 136, 46, 52, 29, 85, 89, 145, 42, 20, 4	736	736, 94, 97, 130, 10, 1
537	537, 83, 73, 58, 89, 145, 42, 20, 4	587	587, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	637	637, 94, 97, 130, 10, 1	687	687, 149, 98, 145, 42, 20, 4	737	737, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4
538	538, 98, 145, 42, 20, 4	588	588, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	638	638, 109, 82, 68, 100, 1	688	688, 164, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	738	738, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
539	539, 115, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	589	589, 170, 50, 25, 29, 85, 89, 145, 42, 20, 4	639	639, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	689	689, 181, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	739	739, 139, 91, 82, 68, 100, 1
540	540, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	590	590, 106, 37, 58, 89, 145, 42, 20, 4	640	640, 52, 29, 85, 89, 145, 42, 20, 4	690	690, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	740	740, 65, 61, 37, 58, 89, 145, 42, 20, 4
541	541, 42, 20, 4	591	591, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4	641	641, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	691	691, 118, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	741	741, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
542	542, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	592	592, 110, 2, 4	642	642, 56, 61, 37, 58, 89, 145, 42, 20, 4	692	692, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	742	742, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
543	543, 50, 25, 29, 85, 89, 145, 42, 20, 4	593	593, 115, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	643	643, 61, 37, 58, 89, 145, 42, 20, 4	693	693, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	743	743, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4
544	544, 57, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	594	594, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	644	644, 68, 100, 1	694	694, 133, 19, 82, 68, 100, 1	744	744, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
545	545, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	595	595, 131, 11, 2, 4	645	645, 77, 98, 145, 42, 20, 4	695	695, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4	745	745, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
546	546, 77, 98, 145, 42, 20, 4	596	596, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4	646	646, 88, 128, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	696	696, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	746	746, 101, 2, 4
547	547, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	597	597, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	647	647, 101, 2, 4	697	697, 166, 73, 58, 89, 145, 42, 20, 4	747	747, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
548	548, 105, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	598	5						

751	751, 75, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	801	801, 65, 61, 37, 58, 89, 145, 42, 20, 4	851	851, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	901	901, 82, 68, 100, 1	951	951, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4
752	752, 78, 113, 11, 2, 4	802	802, 68, 100, 1	852	852, 93, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	902	902, 85, 89, 145, 42, 20, 4	952	952, 110, 2, 4
753	753, 83, 73, 58, 89, 145, 42, 20, 4	803	803, 73, 58, 89, 145, 42, 20, 4	853	853, 98, 145, 42, 20, 4	903	903, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	953	953, 115, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
754	754, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	804	804, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	854	854, 105, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	904	904, 97, 130, 10, 1	954	954, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4
755	755, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	805	805, 89, 145, 42, 20, 4	855	855, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	905	905, 106, 37, 58, 89, 145, 42, 20, 4	955	955, 131, 11, 2, 4
756	756, 110, 2, 4	806	806, 100, 1	856	856, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	906	906, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	956	956, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4
757	757, 123, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	807	807, 113, 11, 2, 4	857	857, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	907	907, 130, 10, 1	957	957, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
758	758, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	808	808, 128, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	858	858, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	908	908, 145, 42, 20, 4	958	958, 170, 50, 25, 29, 85, 89, 145, 42, 20, 4
759	759, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	809	809, 145, 42, 20, 4	859	859, 170, 50, 25, 29, 85, 89, 145, 42, 20, 4	909	909, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	959	959, 187, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
760	760, 85, 89, 145, 42, 20, 4	810	810, 65, 61, 37, 58, 89, 145, 42, 20, 4	860	860, 100, 1	910	910, 82, 68, 100, 1	960	960, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
761	761, 86, 100, 1	811	811, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	861	861, 101, 2, 4	911	911, 83, 73, 58, 89, 145, 42, 20, 4	961	961, 118, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
762	762, 89, 145, 42, 20, 4	812	812, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	862	862, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	912	912, 86, 100, 1	962	962, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
763	763, 94, 97, 130, 10, 1	813	813, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	863	863, 109, 82, 68, 100, 1	913	913, 91, 82, 68, 100, 1	963	963, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
764	764, 101, 2, 4	814	814, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	864	864, 116, 38, 73, 58, 89, 145, 42, 20, 4	914	914, 98, 145, 42, 20, 4	964	964, 133, 19, 82, 68, 100, 1
765	765, 110, 2, 4	815	815, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	865	865, 125, 30, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	915	915, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4	965	965, 142, 21, 5, 25, 29, 85, 89, 145, 42, 20, 4
766	766, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	816	816, 101, 2, 4	866	866, 136, 46, 52, 29, 85, 89, 145, 42, 20, 4	916	916, 118, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	966	966, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4
767	767, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	817	817, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	867	867, 149, 98, 145, 42, 20, 4	917	917, 131, 11, 2, 4	967	967, 166, 73, 58, 89, 145, 42, 20, 4
768	768, 149, 98, 145, 42, 20, 4	818	818, 129, 86, 100, 1	868	868, 164, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	918	918, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	968	968, 181, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
769	769, 166, 73, 58, 89, 145, 42, 20, 4	819	819, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	869	869, 181, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	919	919, 163, 46, 52, 29, 85, 89, 145, 42, 20, 4	969	969, 198, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
770	770, 98, 145, 42, 20, 4	820	820, 68, 100, 1	870	870, 113, 11, 2, 4	920	920, 85, 89, 145, 42, 20, 4	970	970, 130, 10, 1
771	771, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	821	821, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	871	871, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	921	921, 86, 100, 1	971	971, 131, 11, 2, 4
772	772, 102, 5, 25, 29, 85, 89, 145, 42, 20, 4	822	822, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	872	872, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	922	922, 89, 145, 42, 20, 4	972	972, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
773	773, 107, 50, 25, 29, 85, 89, 145, 42, 20, 4	823	823, 77, 98, 145, 42, 20, 4	873	873, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	923	923, 94, 97, 130, 10, 1	973	973, 139, 91, 82, 68, 100, 1
774	774, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	824	824, 84, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	874	874, 129, 86, 100, 1	924	924, 101, 2, 4	974	974, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
775	775, 123, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	825	825, 93, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	875	875, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	925	925, 110, 2, 4	975	975, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
776	776, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	826	826, 104, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	876	876, 149, 98, 145, 42, 20, 4	926	926, 121, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	976	976, 166, 73, 58, 89, 145, 42, 20, 4
777	777, 147, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	827	827, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	877	877, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	927	927, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	977	977, 179, 131, 11, 2, 4
778	778, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	828	828, 132, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	878	878, 177, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	928	928, 149, 98, 145, 42, 20, 4	978	978, 194, 98, 145, 42, 20, 4
779	779, 179, 131, 11, 2, 4	829	829, 149, 98, 145, 42, 20, 4	879	879, 194, 98, 145, 42, 20, 4	929	929, 166, 73, 58, 89, 145, 42, 20, 4	979	979, 211, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
780	780, 113, 11, 2, 4	830	830, 73, 58, 89, 145, 42, 20, 4	880	880, 128, 69, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	930	930, 90, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	980	980, 145, 42, 20, 4
781	781, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	831	831, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	881	881, 129, 86, 100, 1	931	931, 91, 82, 68, 100, 1	981	981, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
782	782, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	832	832, 77, 98, 145, 42, 20, 4	882	882, 132, 14, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	932	932, 94, 97, 130, 10, 1	982	982, 149, 98, 145, 42, 20, 4
783	783, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	833	833, 82, 68, 100, 1	883	883, 137, 59, 106, 37, 58, 89, 145, 42, 20, 4	933	933, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	983	983, 154, 42, 20, 4
784	784, 129, 86, 100, 1	834	834, 89, 145, 42, 20, 4	884	884, 144, 33, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	934	934, 106, 37, 58, 89, 145, 42, 20, 4	984	984, 161, 38, 73, 58, 89, 145, 42, 20, 4
785	785, 138, 74, 65, 61, 37, 58, 89, 145, 42, 20, 4	835	835, 98, 145, 42, 20, 4	885	885, 153, 35, 34, 25, 29, 85, 89, 145, 42, 20, 4	935	935, 115, 27, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	985	985, 170, 50, 25, 29, 85, 89, 145, 42, 20, 4
786	786, 149, 98, 145, 42, 20, 4	836	836, 109, 82, 68, 100, 1	886	886, 164, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	936	936, 126, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	986	986, 181, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
787	787, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	837	837, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	887	887, 177, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	937	937, 139, 91, 82, 68, 100, 1	987	987, 194, 98, 145, 42, 20, 4
788	788, 177, 99, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4	838	838, 137, 59, 106, 37, 58, 89, 145, 42, 20, 4	888	888, 192, 86, 100, 1	938	938, 154, 42, 20, 4	988	988, 209, 85, 89, 145, 42, 20, 4
789	789, 194, 98, 145, 42, 20, 4	839	839, 154, 42, 20, 4	889	889, 209, 85, 89, 145, 42, 20, 4	939	939, 171, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	989	989, 226, 44, 32, 13, 10, 1
790	790, 130, 10, 1	840	840, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	890	890, 145, 42, 20, 4	940	940, 97, 130, 10, 1	990	990, 162, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
791	791, 131, 11, 2, 4	841	841, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	891	891, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	941	941, 98, 145, 42, 20, 4	991	991, 163, 46, 52, 29, 85, 89, 145, 42, 20, 4
792	792, 134, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	842	842, 84, 80, 64, 52, 29, 85, 89, 145, 42, 20, 4	892	892, 149, 98, 145, 42, 20, 4	942	942, 101, 2, 4	992	992, 166, 73, 58, 89, 145, 42, 20, 4
793	793, 139, 91, 82, 68, 100, 1	843	843, 89, 145, 42, 20, 4	893	893, 154, 42, 20, 4	943	943, 106, 37, 58, 89, 145, 42, 20, 4	993	993, 171, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4
794	794, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	844	844, 96, 117, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	894	894, 161, 38, 73, 58, 89, 145, 42, 20, 4	944	944, 113, 11, 2, 4	994	994, 178, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
795	795, 155, 51, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	845	845, 105, 26, 40, 16, 37, 58, 89, 145, 42, 20, 4	895	895, 170, 50, 25, 29, 85, 89, 145, 42, 20, 4	945	945, 122, 9, 81, 65, 61, 37, 58, 89, 145, 42, 20, 4	995	995, 187, 114, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4
796	796, 166, 73, 58, 89, 145, 42, 20, 4	846	846, 116, 38, 73, 58, 89, 145, 42, 20, 4	896	896, 181, 66, 72, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	946	946, 133, 19, 82, 68, 100, 1	996	996, 198, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4
797	797, 179, 131, 11, 2, 4	847	847, 129, 86, 100, 1	897	897, 194, 98, 145, 42, 20, 4	947	947, 146, 53, 34, 25, 29, 85, 89, 145, 42, 20, 4	997	997, 211, 6, 36, 45, 41, 17, 50, 25, 29, 85, 89, 145, 42, 20, 4
798	798, 194, 98, 145, 42, 20, 4	848	848, 144, 33, 18, 65, 61, 37, 58, 89, 145, 42, 20, 4	898	898, 209, 85, 89, 145, 42, 20, 4				

## 2 Bouquet final

Nous sommes à présent en mesure de démontrer le théorème initial.

*Démonstration.* Soit  $n \geq 1$ .

- Si  $n \leq 1000$  alors le résultat voulu est vrai d'après la propriété précédente.
- Si  $n \geq 1000$ , on a vu que la suite atteint nécessairement à un moment donné un nombre inférieur ou égal à 1000. D'après le cas précédent, on peut alors dire qu'on atteindra nécessairement 1 ou 4 à un moment donné.  $\square$